Department of Communications Engineering

Communication Systems

Third Year Class

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Lecture 1

Review of Signals and Systems

* What is a signal? it is the history of something's behaviour.

* Ex. Voltage behaviour through one second, maybe it increases or decreases or to be a zero value.

* The events of increasing or decreasing or any other event happen to voltage or current can be represented by a mathematical model (mathematical equation).

* Generally, triganometric functions could model the interaction of a signal. Such triganometric functions can be sine or cosine.

* The sinusoidal signal can be written generally as

$$x(t) = A \cos(\omega_0 t + \phi) \qquad (1)$$

where:-

A : amplitude of $x(t)$,

ω_0 : is the radian frequency (radian/second),

t : time in seconds,

ϕ : phase in radians,

continue ---

$\omega_0 = 2\pi f_0$ radians/second (rad/s)

$f_0 = \dfrac{\omega_0}{2\pi}$ is the frequency in seconds (s).

* Frequency f_0 : It means the repeatitions of the events of the signal $x(t)$ within one second.

Ex. suppose $x(t) = 3\cos\left[2\pi(2)t - \frac{4\pi}{10}\right]$, $x(t)$ can be drawn as in Figure 1.1

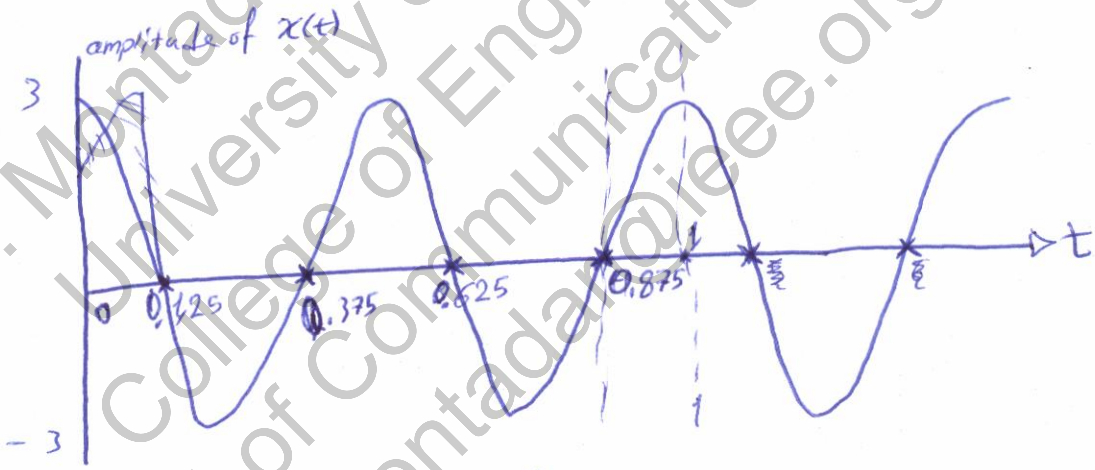


Figure 1.1

* The events repeat every 0.5 second, in other words, there will be two rotations in each one second.

* From your math course, remember the following important properties:

Equivalence

$$\sin(\theta) = \cos\left(\theta - \frac{\pi}{2}\right) \quad \text{———} (2)$$

$$\cos(\theta) = \sin\left(\theta + \frac{\pi}{2}\right) \quad \text{———} (3)$$

periodicity

$$\cos(\theta - 2\pi\ell) = \cos(\theta) \quad \text{———} (4)$$

ℓ is an integer

$$\sin(\theta - 2\pi\ell) = \sin(\theta) \quad \text{———} (5)$$

Evenness:

$$\cos(-\theta) = \cos(\theta) \quad \text{———} (6)$$

oddness

$$\sin(-\theta) = -\sin(\theta)$$

* Also remember these some important identities:

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \text{——} (7)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \quad \text{——} (8)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \quad \text{——} (9)$$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \quad \text{——} (10)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \quad \text{——} (11)$$

$$\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta) \quad \text{——} (12)$$

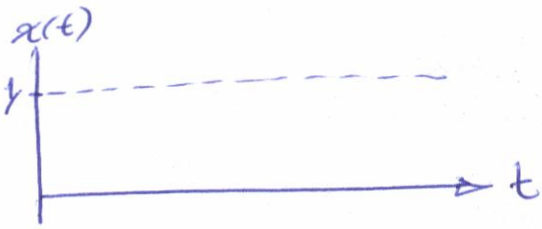
$$\sin^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta) \quad \text{——} (13)$$

* Furthermore Euler formula is

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j} \quad \text{——} (14)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad \text{——} (15)$$

<u>Ex.</u> Let $x(t) = \cos(2\pi(0)t)$, then $x(t)$ can be plotted as



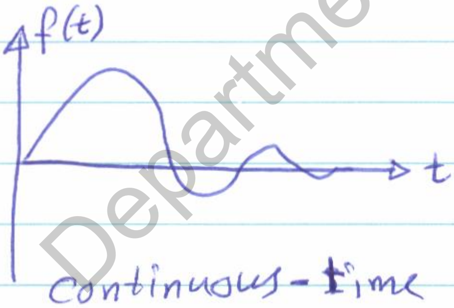
Classification of signals:

* In communication systems, there are different types of signals. To simplify the study of communication systems, signals can be classified as:

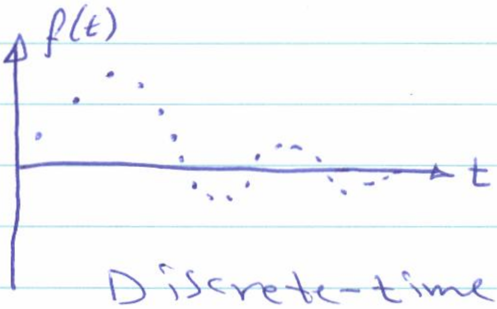
1- Continuous-time and Discrete-time signals,
2- Even and odd signals,
3- periodic and aperiodic (non periodic) signals,
4- Deterministic and random signals,
5- Analog and digital signals,
6- Real and complex signals, and
7- Energy and power signals.

* The signal is said continuous-time if its value is defined for all values of the time variable.

* Discrete signals are those signals which their values are defined at a specified instants of the time.



continuous-time signal



Discrete-time signal.

Even and Odd Components of a Signal

* Every signal consists of two parts, these two parts called even and odd components.

* That is, the signal can be written using the even and odd components.

□ **Evenness**: The function is even, if and only if,

$$f(t) = f(-t) \quad \text{——— (16)}$$

* $f(t)$ has the same value at t & $-t$.
* Hence, $f(t)$ symmetric about the vertical access.

□ **Oddness**: The function is odd, if and only if,

$$f(t) = -f(-t) \quad \text{——— (17)}$$

* The value of the function at time t is the negating value at time $-t$.
* Hence, $f(t)$ is symmetric about the origin.

* In other words :-

even X even = even

even X odd = odd

odd X even = odd

odd X odd = even

* In the communications engineering, even and odd properties simplify the problems very much.

Ex. * if $f(t)$ is even, then

$$\int_{-a}^{a} f(t)\,dt = 2\int_{0}^{a} f(t)\,dt$$

* if $f(t)$ is odd, then

$$\int_{-a}^{a} f(t)\,dt = \text{zero}$$

* Every signal $f(t)$ can be expressed as a sum of its even and odd components as,

$$f(t) = \underbrace{\frac{1}{2}\left[f(t) + f(-t)\right]}_{\text{even component}} + \underbrace{\frac{1}{2}\left[f(t) - f(-t)\right]}_{\text{odd component}} \quad (18)$$

~~From equation (18), the even component is~~

or

$$f(t) = f_e(t) + f_o(t) \quad (19)$$

where

$$f_e(t) = \frac{1}{2}\left[f(t) + f(-t)\right] \quad (20)$$

and

$$f_o(t) = \frac{1}{2}\left[f(t) - f(-t)\right] \quad (21)$$

<u>EX.</u> What are the even and odd components of e^{jt}?

<u>Ans</u>
$$e^{jt} = f_e(t) + f_o(t)$$

$$f_e(t) = \frac{1}{2}\left[e^{jt} + e^{-jt}\right] = \cos(t)$$

$$f_o(t) = \frac{1}{2}\left[e^{jt} - e^{-jt}\right] = j\sin(t)$$

* Periodicity of a signal :

* periodic signal stands for a signal that repeates itself ~~at~~ each a specified time period.

* The signal $f(t)$ can be considered as a periodic signal if and only if

$$f(t) = f(t + T) \text{ ——— } (22)$$

where T is called the period of the signal $f(t)$. In other words, $f(t)$ repeats itself every T seconds.

T is the fundamental period.

* If $g(t) = f_1(t) + f_2(t)$ — (23)

Let $f_1(t) = f_1(t + T_1)$ — (24) {periodic with period T_1}

$f_2(t) = f_2(t + T_2)$ — (25) {Periodic with period T_2}

then $g(t)$ in equation (23) is periodic if and only if

$$\boxed{\frac{T_1}{T_2} = n = \text{rational number}} \text{ ——— } (26)$$

* The last equation informs us that T_1 can be calculated using T_2,

$$\boxed{T_1 = n T_2} \qquad (27)$$

or we can calculate T_2 from T_1 as

$$T_2 = \frac{T_1}{n} \qquad (28)$$

$$\boxed{T_0 = n T_2 = T_1} \qquad (29)$$

EX. given $y(t) = \cos(t + \pi/4)$,

$y(t) = \cos(\omega_0 t + \theta)$

$\therefore \omega_0 = 1 = 2\pi f_0$

hence $f_0 = \frac{1}{2\pi} \Rightarrow T_0 = \frac{1}{f_0} = 2\pi$ seconds

EX. $x(t) = \sin\left(\frac{2\pi}{5} t\right)$,

$x(t) = \sin(\omega_0 t + \theta) \Rightarrow \omega_0 = \frac{\cancel{2\pi}}{5} = \frac{\cancel{2\pi}}{T_0} \Rightarrow T_0 = 5$ sec.

T_0 is 5 seconds (the fundamental period).

EX. Assume $g(t) = \sin(\frac{\pi}{2}t) + \cos(\frac{\pi}{4}t)$, is $g(t)$ periodic? if so, what is the fundamental period?

solution $g(t) = g_1(t) + g_2(t)$

$$g_1(t) = \sin(\frac{\pi}{2}t) = \sin(\omega_1 t)$$

$$\therefore \omega_1 = \frac{\pi}{2} = \frac{2\pi}{T_1} \Rightarrow \boxed{T_1 = 4\ s}$$

$$g_2(t) = \cos(\frac{\pi}{4}t) = \cos(\omega_2 t)$$

$$\therefore \omega_2 = \frac{\pi}{4} = \frac{2\pi}{T_2} \Rightarrow \boxed{T_2 = 8\ s}$$

$$\frac{T_1}{T_2} = \frac{4}{8} = \frac{1}{2} = 0.5 \text{ (rational number)}$$

$\therefore$ $g(t)$ is periodic with a fundamental period T_0,

$$T_0 = 2T_1 = T_2 = 8\ s.$$

EX. $y(t)=\cos^2(t)$, is it periodic? what is the fundamental period T_0?

solution yes, $y(t)$ is periodic

$$y(t)=\frac{1}{2}+\frac{1}{2}\cos(2t)=y_1(t)+y_2(t)$$

$y_1(t)=\frac{1}{2}$ is a D.C. with arbitrary period

$y_2(t)=\cos(2t)$ is periodic with period T_2,

$$T_2=\frac{2\pi}{\omega_1}=\frac{2\pi}{2}=\pi \text{ s}$$

$\therefore$ $y(t)$ is periodic with fundamental period $T_0=\pi$ s.

EX. $y(t)=\sin(t)+\cos(\sqrt{3}\,t)$, is $y(t)$ periodic? if so, what is the fundamental period T_0?

solution $y(t)=y_1(t)+y_2(t) \Rightarrow y_1(t)=\sin(t) \Rightarrow T_1=\frac{2\pi}{\omega_1} \Rightarrow$

$$T_1=\frac{2\pi}{1}=2\pi \text{ s.}$$

$$T_2=\frac{2\pi}{\omega_2}=\frac{2\pi}{\sqrt{3}} \text{ s.}$$

$$\frac{T_1}{T_2}=\frac{2\pi}{\frac{2\pi}{\sqrt{3}}}=2\pi\,\frac{\sqrt{3}}{2\pi}=\sqrt{3} \text{ s.}$$

$\sqrt{3}$ is not a rational number, hence $y(t)$ is not periodic.

* In the following subjects, you will need to keep in mind the following:-

$$\int_0^T \sin(n\omega t)\,dt = 0 \quad (30)$$

$$\int_0^T \cos(n\omega t)\,dt = 0 \quad (31)$$

$$\int_0^T \cos(m\omega t)\sin(n\omega t)\,dt = 0 \quad (32)$$

where n and m are integers.

* Furthermore,

$$\int_0^T \sin(n\omega t)\sin(m\omega t)\,dt = \begin{cases} 0 & m \neq n \\ \frac{T}{2} & m = n \end{cases} \quad (33)$$

$$\int_0^T \cos(n\omega t)\cos(m\omega t)\,dt = \begin{cases} 0 & m \neq n \\ \frac{T}{2} & m = n \end{cases} \quad (34)$$

* moreover,

$$\cos(n\pi) = (-1)^n \quad (35)$$

$$\sin(n\pi) = 0 \quad (36)$$

$$\cos\left(n\frac{\pi}{2}\right) = 0 \quad \text{when } n \text{ is odd} \quad (37)$$